1. A crime is committed by one of two suspects, A and B. Initially, there is equal evidence against both of them. In further investigation at the crime scene, it is found that the guilty party had a blood type found in 10% of the population. Suspect A does match this blood type, whereas the blood type of Suspect B is unknown. (a) Given this new information, what is the probability that A is the guilty party? (b) Given this new information, what is the probability that B’s blood type matches that found at the crime scene?

(a) Let's denote G as the event that A is guilty and M as the event that the blood type matches the one found at the crime scene. We need to find P(G | M), the probability that A is guilty given that the blood type matches.

According to Bayes' theorem:

P(G | M) = (P(M | G) \* P(G)) / P(M)

P(M | G) is the probability that the blood type matches given that A is guilty, which is 1 (100%) since A does match the blood type.

P(G) is the initial probability that A is guilty, which is 0.5 (equal evidence against both suspects initially).

P(M) is the probability that the blood type matches, which can be calculated using the law of total probability:

P(M) = P(M | G) \* P(G) + P(M | not-G) \* P(not-G)

P(M | not-G) is the probability that the blood type matches given that A is not guilty, which is 0.1 (10% of the population has the blood type).

P(not-G) is the initial probability that A is not guilty, which is also 0.5 (equal evidence against both suspects initially).

Now we can substitute the values into the equation:

P(M) = (1 \* 0.5) + (0.1 \* 0.5) = 0.55

Finally, we can calculate P(G | M):

P(G | M) = (1 \* 0.5) / 0.55 ≈ 0.9091

Therefore, the probability that A is guilty given that the blood type matches is approximately 0.9091, or 90.91%.

(b) Let's denote B as the event that B's blood type matches the one found at the crime scene. We need to find P(B | M), the probability that B's blood type matches given that the blood type matches.

According to the law of total probability:

P(B | M) = P(B and M) / P(M)

P(B and M) is the probability that B's blood type matches and the blood type matches, which is P(B) \* P(M | B). Since B's blood type is unknown, we assume it is independent of the crime and has the same probability as the population, which is 0.1 (10%).

P(M) is the probability that the blood type matches, which we calculated as 0.55 in part (a).

Now we can substitute the values into the equation:

P(B | M) = (0.1 \* P(M | B)) / 0.55

Since the blood type matches with a probability of 0.1 regardless of whether B is guilty or not, P(M | B) = 1.

P(B | M) = (0.1 \* 1) / 0.55 ≈ 0.1818